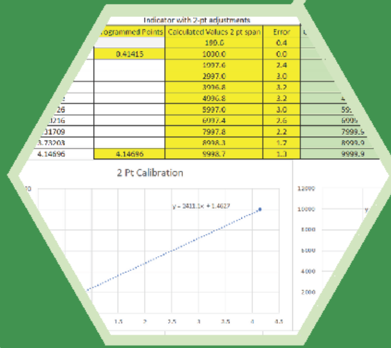
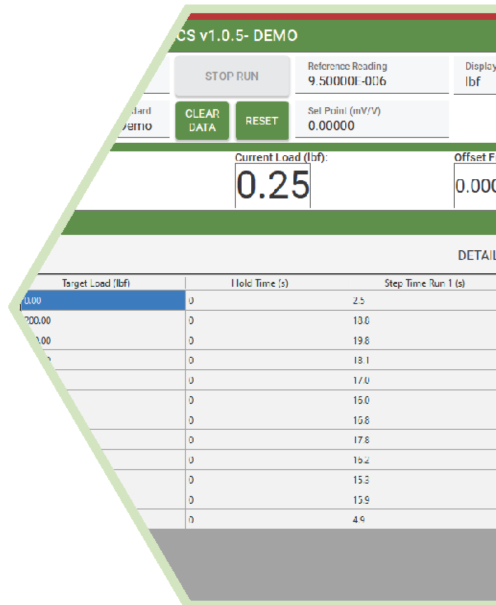


Morehouse Force Calculation Guidance

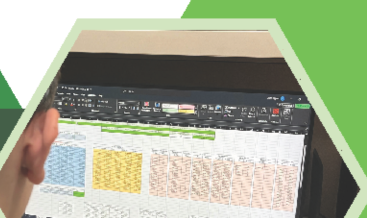
Morehouse Calculations found on Certificates of Calibration



Written By

Henry Zumbrun & Zach Shearer

Morehouse Instrument Company



Morehouse Calculations found on Certificates of Calibration

Table of Contents

Introduction

Not all calculations are straightforward on certificates of calibration, and different companies use different math equations, resulting in different results.

This document aims to share some of the common math equations behind the calculations on Morehouse calibration certificates.

Standards like ASTM E74 and ISO 376 provide detailed information on how those calculations are performed. Therefore, the guidance on those is limited.

Standard Calculations for Verification of Load Cell Specifications

Load cell specification sheet.

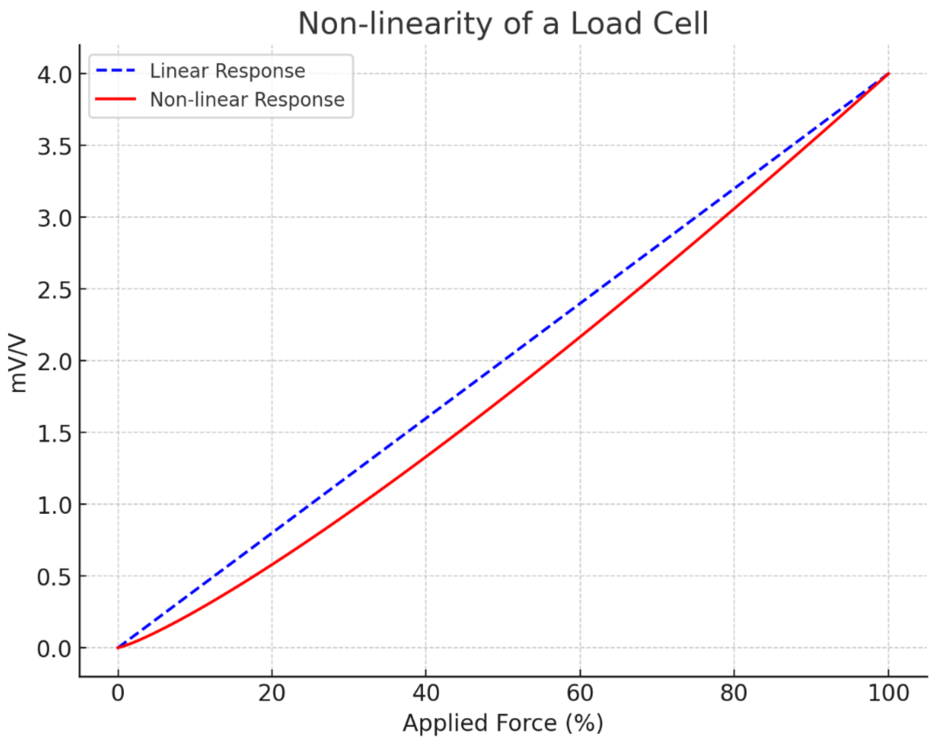
Specifications	Model - Capacity (lbf / kN)					
	300-2K / 1-10	5K-10K / 20-50	25K-50K /100-250	60K / 300	100K / 500	200K / 900
Accuracy						
Static Error Band, % R.O.	±0.02	± 0.03	± 0.03	± 0.03	± 0.05	± 0.05
Non-Linearity, % R.O.	±0.02	± 0.03	± 0.03	± 0.03	± 0.05	± 0.05
Hysteresis, % R.O.	± 0.02	± 0.04	± 0.04	± 0.04	± 0.05	± 0.05
Non-Repeatability, % R.O.	± 0.005	± 0.005	± 0.005	± 0.005	± 0.005	± 0.005
Creep, % Rdg / 20 Min.	± 0.015	± 0.015	± 0.015	± 0.015	± 0.015	± 0.015
Off-Center Load Sensitivity, %/in	±0.1	± 0.1	± 0.1	± 0.1	± 0.1	± 0.1
Side Load Sensitivity, %	± 0.1	± 0.1	± 0.1	± 0.1	± 0.1	± 0.1
Zero Balance, % R.O.	± 1.0	± 1.0	± 1.0	± 1.0	± 1.0	± 1.0

Non-Linearity Calculations

Definition

The quality of a function that expresses a relationship that is not one of direct proportion. For force measurements, **Non-Linearity** is defined as the algebraic difference between the output at a specific load and the corresponding point on the straight line drawn between the outputs at minimum load and maximum load. It is usually calculated between 40 - 60 % of the full scale.

An ideal measurement device has a perfectly linear response to force applied ratio. However, this is rarely true; most devices have a non-linear ratio. The purpose of the non-linearity calculation is to show how the recorded responses deviate from the ideal ratio. Non-linearity is typically expressed in the percent of full-scale (% FS).



A line between the initial zero and full-scale points should be drawn to calculate Non-Linearity. This line represents the ideal response ratio that is compared against each of the ascending points. Calculate the slope using the equations below to draw a line between the two points. With the slope, the intercept can be calculated using either of the two points used to calculate the slope with the equation below. With the line properties calculated, use each of the recorded responses in the final calculation below to calculate the % FS of non-linearity.

Shortcomings

Non-Linearity is a great way to visualize how much a measuring device deviates from an "ideal" device. However, all points may be perfectly linear, but if the full-scale point is non-linear itself, the rest of the points will appear to be non-linear.

Some manufacturers use higher-order equations to improve their Non-Linearity specification. Therefore, it is important to ask them how they calculate non-linearity.

At Morehouse, we use the more conservative straight-line approach method.

Calculate Slope

Slope = (0$_{start(force)}$ – FullScale$_{(force)}$) / (0$_{start(response)}$ – FullScale$_{(response)}$)

Calculate Intercept

Intercept = FullScale$_{(force)}$ – Slope x FullScale$_{(response)}$

Calculate Non-Linearity per Response

Non-Linearity = (Point$_{(force)}$ – (Slope x Point$_{(response)}$ + Intercept)) / FullScale$_{(force)}$

For example, a load cell reads 0 at 0 lbf, 1.20003 at 600 lbf, and 2.00010 mV/V at 1000 lbf.

To calculate the slope, the formula would be (0-1000)/(0-2.00010) = 499.975001249937

To calculate the Intercept 1000 – (499.975001249937 * 2.00010) = 0.0

Non-Linearity = (600 – (499.975001249937*1.20003+0))/1000 = 0.0000150

This value is 0.0015 % using the 600 lbf (60 %) point.

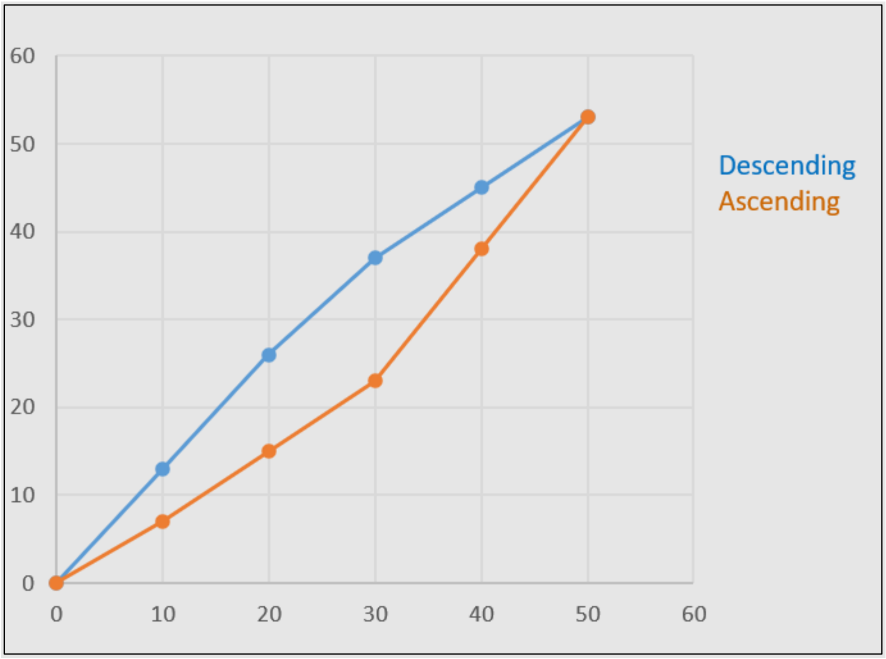
Hysteresis Calculations

Definition

Hysteresis: The phenomenon in which the value of a physical property lags changes in the effect causing it. An example is when magnetic induction lags the magnetizing force.

For force measurements, **Hysteresis** is defined as the difference between two responses of a single given load, one ascending from the lowest non-zero load applied, the other descending from the full-scale load. Hysteresis is typically calculated at a 40 % load. The purpose of calculating Hysteresis is to identify how well the materials of the device recover after being fully loaded. Hysteresis is typically expressed as a positive value and in percent of full-scale (% FS).

To calculate Hysteresis, two responses must be recorded for the same load applied. Following the equation below, the descending response is subtracted from the ascending response (or vice versa). The absolute difference is then divided by the full-scale response. To ensure Hysteresis is a positive value, the absolute value of the quotient is used.



Calculate Hysteresis

Hysteresis = | (Ascending(response) - Descending(response)) / FullScale (response) |

Shortcomings

Errors from Hysteresis can be high enough that if a load cell is used to make descending measurements, then it must be calibrated with a descending range. That range is only valid if the load cell is loaded to the highest force point in the range before descending measurements are made.

The difference in output on an ascending curve versus a descending curve can be significant. For example, an exceptionally good Morehouse 100K precision shear-web load cell had an output of -2.03040 on the ascending curve and -2.03126 on the descending curve. Using the ascending-only curve would result in an additional error of 0.042 %.

At Morehouse, our calibration lab sampled several instruments and recorded the following differences.

Load Cell Manufacturer (names removed)	1	2	3	4	5	5	3	4
Ascending Output 50 % Force Point	1.49906	1.20891	-2.0304	24990	-5.18046	-2.49899	-2.0886	-2.15449
Descending Output 50 % Force Point	1.49947	1.21022	-2.03126	25020	-5.18265	-2.50103	-2.08846	-2.15579
Difference	0.027%	0.108%	0.042%	0.120%	0.042%	0.082%	0.007%	0.060%

Load cells from five different manufacturers were sampled, and the results were recorded. The differences between the ascending and descending points varied from 0.007 % (shear web type cell) to 0.120 % on a column type cell. On average, the difference was approximately 0.06 %. Six of the seven tests were performed using deadweight primary standards at Morehouse, which are accurate within 0.002 % of the applied force.

Non-Repeatability Calculations

Non-Repeatability: The maximum difference between output readings for repeated loadings under identical loading and environmental conditions. Usually, this is expressed in units as a % of rated output (RO). Non-repeatability tells the user a lot about the performance of the load cell. It is important to note that non-repeatability does not tell the user about the load cell's reproducibility or how it will perform under different loading conditions (randomizing the loading conditions). At Morehouse, we have observed numerous load cells with good non-repeatability specifications that do not perform well when the loading conditions are randomized or the load cell is rotated 120 degrees as required by ISO 376 and ASTM E74.

The calculation of non-repeatability is straightforward. First, compare each observed force point's output and run a difference between those points. The formula would look like this: *Non-repeatability = ABS(Run1-Run2)/AVERAGE (Run1, Run2, Run3) *100.* Do this for each combination or runs, and then take the maximum of the three calculations.

non-repeatability calclulations		
Run 1	Run 2	Run 3
4.0261	4.02576	4.02559
Difference b/w 1 & 2 (%FS)	Difference b/w 1 & 3 (%FS)	Difference b/w 2 & 3 (%FS)
0.0084	0.0127	0.0042
Non-Repeatability (%FS)=		0.013

non-repeatability calclulations		
Run 1	Run 2	Run 3
4.0261	4.02576	4.02559
Difference b/w 1 & 2 (%FS)	Difference b/w 1 & 3 (%FS)	Difference b/w 2 & 3 (%FS)
=ABS(U4-V4)/AVERAGE($U$4:$W$4)*100	=ABS(U4-W4)/AVERAGE($U$4:$W$4)*100	=ABS(W4-V4)/AVERAGE($U$4:$W$4)*100
Non-Repeatability (%FS)=		=MAX(U9:W9)

Static Error Band (SEB)

Static Error Band: The band of maximum deviations of the ascending and descending calibration points from a best-fit line through zero output. It includes the effects of Non-Linearity, Hysteresis, and non-return to minimum load. It is usually expressed in units of % of full scale.

If the load cell is always used to make ascending and descending measurements, this term best describes the load cell's actual error from the straight line drawn between the ascending and descending curves.

Calculate SEB

Our goal is to find a line that results in the smallest maximum error. This line also needs to fit through the origin (0, 0), so only the slope needs to be calculated via $(y_1+y_2) / (x_1+x_2)$. The best approach to this is to iterate across every pair of percent force applied of full scale (**% FS**) and the zero-adjusted responses.

For each pair, calculate the slope, use the slope to calculate the percent error for all % FS, and take the largest error as that slope's "absolute error" value. Repeat this for all possibilities, taking the slope that has the smallest absolute error value.

Excel Macro Snippet

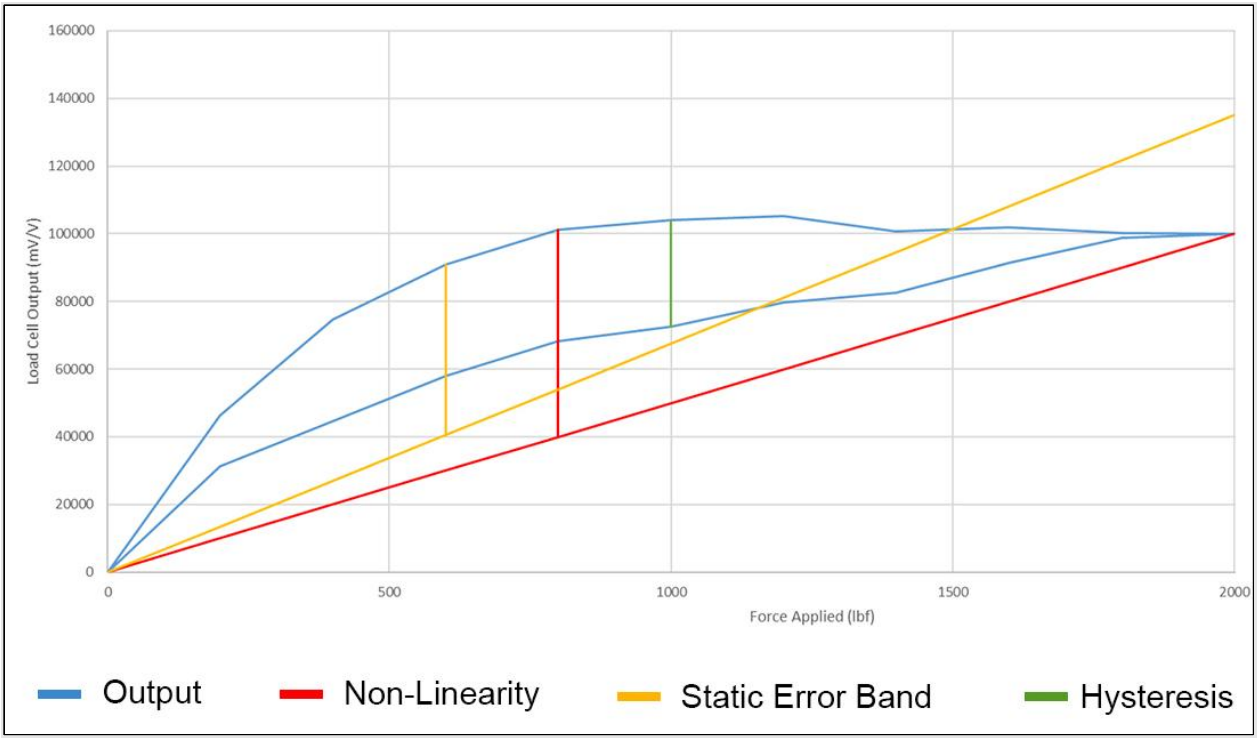
```
'Iterate across every permutation of 2 points
For i = 0 To N - 1
  'Start at i + 1 to duplicating work, reducing iterations
  For j = i + 1 To N - 1
    'Prevent checking the same point and dividing by zero
    If i <> j And PercentFS(i) + PercentFS(j) <> 0 Then
      'tempSlope = (Vj + Vi) / (Rj + Ri)
      maxError = 0
      tempSlope = (Responses(j + 2, 1) + Responses(i + 2, 1)) / (PercentFS(j) + PercentFS(i))

      'Ensure we don't accidentally set the minimum error to 0 or divide by 0
      If tempSlope <> 0 Then
        For k = 0 To N - 1
          tempError = (Responses(k + 2, 1) - tempSlope * PercentFS(k)) / tempSlope

          'Take the largest error for this slope
          If Abs(tempError) > Abs(maxError) Then
            maxError = tempError
            slope = tempSlope
          End If
        Next k

        'Find the slope that provides the lowest maximum error
        If IsNull(minError) Or Abs(maxError) < Abs(minError) Then
          minError = maxError
          sebSlope = slope
        End If
      End If
    End If
  Next j
Next i
```

Shortcomings

If the load cell is used for ascending measurements and, on occasion, descending measurements are needed, the user may want to evaluate Non-Linearity and Hysteresis separately, as those two definitions may provide a more accurate depiction of the load cell's performance.

What needs to be avoided is a situation where a load cell is calibrated following a standard such as ASTM E74 or ISO 376 and additional uncertainty contributors for Non-Linearity and Hysteresis are added. ASTM E74 has a procedure and calculations that, when followed, use a method of least squares to fit a polynomial function to the data points. The standard uses a specific term called the Lower Limit Factor (LLF), which is a statistical estimate of the error in forces computed from a force-measuring instrument's calibration equation when the instrument is calibrated following the ASTM E74 practice.

Summary Graph of Terms



Resolution

Definition

The smallest change in a quantity being measured causes a perceptible change in the corresponding indication.

When an instrument is new and submitted to Morehouse, we try to maximize the resolution to at least 100,000 counts.

Most systems we recommend are mV/V and follow this simple formula when resolution is reported.

*Resolution = (Force Applied / Output of the Instrument at that force) * readability of instrument*

Note: We do this for every point and take the average of all points.

Example:

At 5,000 lbf, the instrument reads 1.99691 mV/V and counts by 0.00001 mV/V

Resolution = (5,000/1.99691) * 0.00001

Resolution = 2503.86877 * 0.00001

Resolution = 0.025 lbf

Interpolated Zero (Method B)

Definition

During the process of a calibration, creep, and deflection are introduced into the responses of the material. The ASTM E-74 and ISO 376 standards outline approaches to best address these and compensate for the starting zero. While numerous approaches exist, we opt for one that provides a symmetrical distribution and emphasizes on time being the main contributor to creep. It is worth noting that since many equations use this adjusted data, a different approach will cause different results from subsequent calculations.

The equation below is used to calculate the adjusted responses. The difference of the ending zero and the starting zero responses is multiplied against a ratio. This ratio is calculated between the point's position in the force series and the number of time intervals in the force series (from one response to the next). All of this is added to the response of the starting zero, which is then subtracted from a point's response.

Ratio Explained

The ratio is specifically the percentage of the ending zero's response's effect on the point's response. To clarify, both zeroes' responses apply a percentage of their response to a given point's response. The first non-zero point in the segment uses only the beginning zero (fraction = 0), while the last non-zero point uses only the ending zero (fraction = 1). Points between get a linearly interpolated fraction.

If the force series contains four non-zero responses, the first non-zero response would have a fraction of 0 % (0/3), the second 33.3 % (1/3), the third 66.7 % (2/3), and the fourth 100 % (3/3). Position is zero-indexed from 0 to n−1, where n is the number of non-zero data points in the segment. A series of forces includes a beginning zero, an ending zero, and every response between. Note: For segments containing a single non-zero data point (n = 1), fraction = 0.5 (the interpolated zero is the average of the two bracketing zero readings). For segments containing both ascending and descending data, fractions are computed independently within each direction sub-segment.

Calculate Adjusted Response (Method B)

Adjusted Response $= Point_{(response)} - (0_{start(response)} + (0_{end(response)} - 0_{start(response)}) \times Point_{(position)} / (n - 1))$

Temperature Correction on Non-Compensated Devices

For devices that are not temperature-compensated.

ASTM E74 has detailed guidance on temperature corrections for correcting the temperature for uncorrected devices at a different temperature than the temperature at which the device was calibrated. The correction involves correcting the force value for temperature by reducing it by 0.027 % for every 1 °C by which the ambient temperature exceeds the temperature of calibration. When the ambient temperature falls below the calibration temperature, the measured force value should be adjusted upward by an appropriate amount.

When Morehouse calibrates any instrument in our force laboratory, we maintain our temperature to about 23.0 °C ± 1.0 °C (Typically 23.0 °C ± 0.5 °C).

However, when we calibrate Morehouse Proving Rings, we do monitor the temperature of the ring and correct for temperature. The formula we use corrects the reading based on the recorded temperature when zero readings are taken.

When we perform a full calibration, we interpolate the temperature change between zeros using startTemp + ((endTemp - startTemp) * (position + 0.5) / n).

The correction takes the zero corrected value (Normalized Data) and divides by the correction formula.

Normalized Value/(1-.00027)*(Interporlated temperature - 23)

Expanded Uncertainty Per Point Non-ASTM

Definition

Per ILAC P-14:09/2020 section 5.4 Contributions to the uncertainty stated on the calibration certificate shall include relevant short-term contributions during calibration and contributions that can reasonably be attributed to the customer's device. Where applicable the uncertainty shall cover the same contributions to uncertainty that were included in evaluation of the CMC uncertainty component, except that uncertainty components evaluated for the best existing device shall be replaced with those of the customer's device. Therefore, reported uncertainties tend to be larger than the uncertainty covered by the CMC.

The simple equation we use to report the Expanded Uncertainty Per Point is:

$$\mathbf{2\ x\ k_{95\%}}\left(\sqrt{\left(\frac{\mathbf{CMC\ Force\ Standard(s)}}{\mathbf{k_{CMC}}}\right)^2+\left(\frac{\mathbf{Resolution_{UUT}}}{\sqrt{\mathbf{12}}}\right)^2+\left(\frac{\mathbf{Repeatability_{UUT}}}{\mathbf{1}}\right)^2+\left(\frac{\mathbf{CMC\ (DMM)}}{\mathbf{k_{CMC}}}\right)^2+\cdots(\mathbf{u}_{Other})^2}\right)$$

The calculation will vary depending on the number of standards used, such as Deadweight and a DMM.

Thus, we created a spreadsheet to help anyone recreate these formulas.

https://mhforce.com/documentation-tools/?_sft_support-item-tag=spreadsheet-tool

Note: Often, the repeatability of the UUT is captured in our CMC of the best existing device. Not every item submitted has multiple data points; thus, the reported Expanded Measurement Uncertainty does not include repeatability.

Expanded Uncertainty Per Point ASTM

This Expanded measurement uncertainty equation is used.

$$2 \text{ x } k_{95\%}\left(\sqrt{\left(\frac{\text{CMC Force Standard(s)}}{k_{CMC}}\right)^2+\left(\frac{\text{Resolution}_{UUT}}{\sqrt{12}}\right)^2+\left(\frac{\text{ASTM LLF}_{UUT}}{2.4}\right)^2+\left(\frac{\text{CMC (DMM)}}{k_{CMC}}\right)^2+\cdots(u_{Other})^2}\right)$$

The calculation will vary depending on the number of standards used such as Deadweight and a DMM.

Thus, we created a spreadsheet to help anyone recreate these formulas.

https://mhforce.com/documentation-tools/?_sft_support-item-tag=spreadsheet-tool

Calibration Coefficients (Polynomial Equation)

Definition

While ideally, a measurement device will respond in a perfectly straight line, realistically, its responses tend to follow a slight curve. To represent the device's response curve, coefficients are derived from the adjusted data after interpolating zeros or subtracting the initial zero tare. These adjusted responses are then typically used in the least squares method to calculate coefficients.

Additionally, various methods exist for calculating the best degree of fit for the coefficients to fit the responses as closely as possible using the smallest degree of fit. These methods are outside the scope of this document but are covered in various standards and can be readily found online. The goal is to use the curve to predict responses or forces given a force or response, respectively.

Calculate Coefficients

Using the forces as ***x*** and the responses as ***y*** will generate coefficients for calculating a response given a specific force (A Coefficients). Likewise, using responses as ***x*** and forces as ***y*** will create coefficients for calculating a force given a specific response (B Coefficients).

Due to the complexity of the equation, it is not shown here. However, the ***LINEST*** function in Excel uses the method of least squares. To perform a higher degree than 1, the ***x*** values can be raised to a sequence of degrees. For example, to obtain the A coefficients of a 4^{th}-degree polynomial, the following formula can be used. Note, that the coefficients are output from left to right with the highest degree on the left.

=LINEST(Responses, Forces^{1,2,3,4})

Supporting multiple runs may require using the ***TOCOL*** function to stack the responses vertically on top of each other. Likewise, the forces may need to be stacked vertically as well.

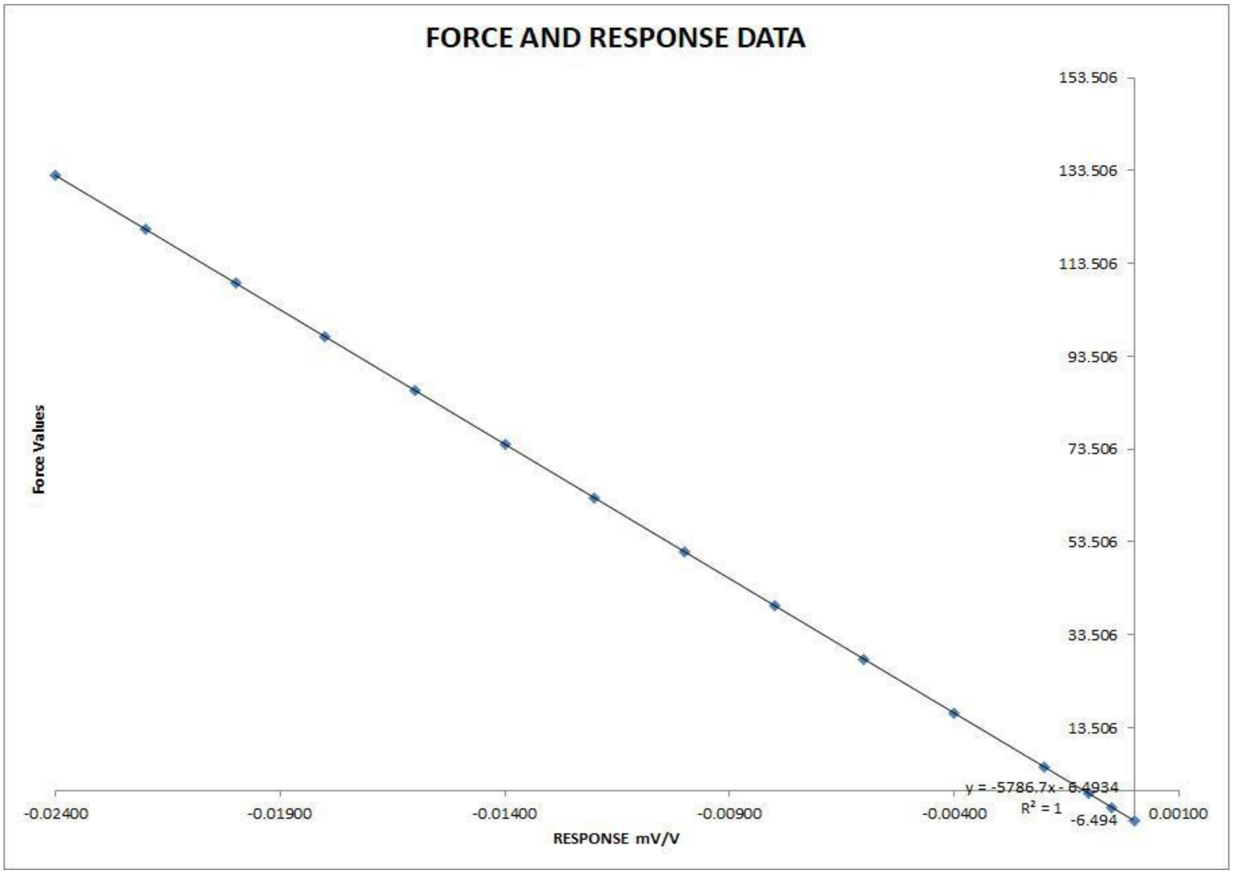
=LINEST(TOCOL(Run1(responses), Run2(responses), Run3(responses)), TOCOL(Forces, Forces, Forces)^{1,2,3,4})

If not using Excel, equations exist online to aid in calculating the coefficients by other means.

Note: The coefficients may differ when using different methods, such as rounding and zero reduction. We typically only care that the coefficients return the same result when the force is applied within the instrument's resolution. If the zero reduction and rounding are the same, the values from the fitted curve should match.

A0 and BO mean that when the recorded units of the instrument are zeroed, or tare is pressed, there will likely be a force reading.

A0 and B0 denote the constants associated with the y-intercept, marking the point where the equation intersects with the y-intercept.



Many end-users do not like this data because they want to see 0 displayed on a device when they 0 the instrument. However, that is not how math works.

In this example, when the meter reads 0, the force value will be -6.49365 lbf or sometimes a rounded number, which would be -6.5 lbf as B_0 = -6.49356. This is because of the way the polynomial equation works in that Force (lbf) is equal to $B_0 + B_1$ * (Response)+B_2 * (Response^2). Simply put, B_1, B_2, and higher are multiplied by the 0 on the meter, except for the first one. The meter will read 0.0 when the Response equals A_0 or -0.00112 mV/V.

In Summary:

- **Zeroing or taring** the instrument sets the response to a specific value (like A0 = -0.00112 mV/V), not necessarily zero.
- The **calibration equation** includes a constant (B0), which means the force isn't zero when the response is zero.
- The force will only be exactly zero when the response equals **A0**, which is a small, non-zero value.

Users may expect to see "0 lbf" after taring, but the instrument is displaying force based on how it's actually calibrated, not just based on expectations.

ASTM Lower Limit Factor

Definition

The ASTM LLF is a statistical estimate of the error in forces computed from the calibration equation of a force–measuring instrument when the instrument is calibrated in accordance with the ASTM E74 standard.

For ASTM LLF, we follow section 8, Calculation and Analysis of Data, from ASTM E74.

Coefficients are used to compare the fitted curve against the actual readings per point. A standard deviation from the differences between the individual values observed in the calibration and the corresponding values taken from the calibration equation, as follows:

$$s_m = \sqrt{\frac{d_1{}^2 + d_2{}^2 + \ldots + d_n{}^2}{n - m - 1}}$$

Where:

d_1, d_2, etc. = differences between the fitted curve and the *n* observed values from the calibration data,

n = number of deflection values, and

m = the degree of the polynomial fit.

Then, the LLF is calculated by converting the standard deviation into force units and multiplying that by 2.4. When the output units are the same as the force units (e.g., both in lbf), the standard deviation is already in force units. When the output units differ from force units (e.g., mV/V output vs. N force), the conversion is: LLF (force) = 2.4 × S / $|B_1|$, where B_1 is the linear coefficient of the polynomial equation.

ASTM Class A and AA verified range of forces.

The ASTM Class A verified range of force is calculated by multiplying the LLF by 400. If this number is lower than the first non-zero force point, the first non-zero force point is used.

The ASTM Class AA verified range of force is calculated by multiplying the LLF by 2000. If this number is lower than the first non-zero force point, the first non-zero force point is used.

ISO 376 Calculations

All our ISO calculations are directly from the standard.

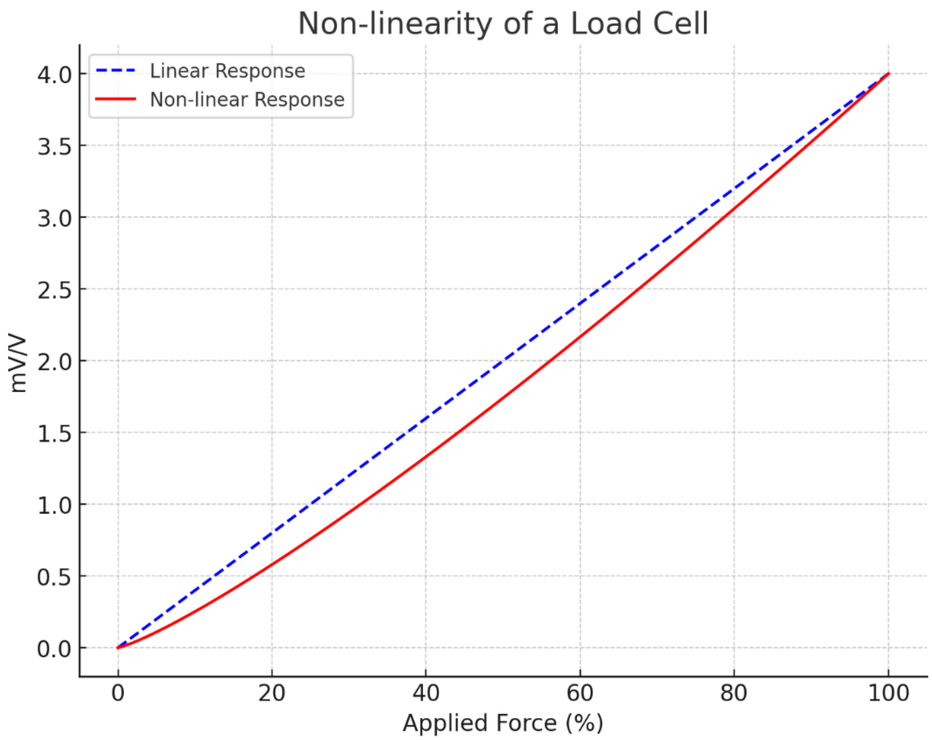
We use a formula listed under the Calculate Coefficients heading, up to a third degree, for ISO 376.

On ISO, we fit a curve to the MU and then report both the minimum uncertainty that can be used (which is the minimum uncertainty that we calculate) and the maximum fit error of the curve. This is the maximum difference between the fitted curve we report for MU and the calculated MU numbers.

The formula for the reported uncertainty follows the Annex C1.10.2 recommendations.

Using a Polynomial Equation versus Linearization via Span Points

Programming a load cell system via span points
Most indicators will allow the end-user to span or capture data points. Several indicators offer many ways of programming points; most will use some linear equation to display the non-programmed points along the curve or line.



Load Cell Curve Versus a Straight Line.

When drawing a straight line between two points, you need to know the slope of the line to predict other points along the line. The common formula is y = mx + b, where m designates the slope of the line, and b is the y-intercept. When programming the indicator using either calibrated points or capturing live readings (data points), the main issue with this approach is that both the indicator and load cell will have some deviations from the straight line. Most indicators have very good linearity and are often better than 0.005 % of the full scale. At the same time, most load cells can be the larger source of error with nonlinear behavior from 0.02 – 1.0 % of full scale or greater.

Of course, factors such as stability, thermal effects, creep recovery, return, and the loading conditions when the points are captured will influence the results.

Programming an indicator via span points will follow a linear approach; some will have a 2-pt span, some 5-pts, and some even more. This method may include a straight line through all the points or several segmented lines. In all cases, there will be additional bias created by this method because the force-measuring system will always have some non-linear behavior.

Note: The segmented line approach with multiple span points will typically help linearize the system and have fewer errors than two span points. However, do not assume multiple span points mean multiple segmented lines.

		Indicator with 2-pt adjustments		
Applied Force lbf	Actual Readings (mV/V)	Programmed Points	Calculated Values 2 pt span	Error
200	0.08279		199.6	0.4
1000	0.41415	0.41415	998.6	1.4
2000	0.82851		1997.6	2.4
3000	1.24302		2997.0	3.0
4000	1.65767		3996.8	3.2
5000	2.07242		4996.8	3.2
6000	2.48726		5997.0	3.0
7000	2.90216		6997.4	2.6
8000	3.31709		7997.8	2.2
9000	3.73203		8998.3	1.7
10000	4.14696	4.14696	9998.7	1.3

Programming an Indicator with a 2-pt Span Calibration.

The figure above exemplifies a Morehouse Calibration Shear Web Load Cell with a Non-Linearity specification of better than 0.05 % of full scale. In this example, the actual non-linearity is about 0.031 %.

Using mV/V values and 0.032 % when using calculated values, it is well below the specification. However, the device cannot claim to be accurate to 0.032 % as this is a short-term accuracy achieved under ideal conditions.

Often, an end-user will see the results above, claim the system is accurate to a number such as 0.05 %, and believe they will maintain it. However, the end-user must account for additional error sources such as stability/drift, reference standard uncertainty that was used to perform the calibration, resolution of the force-measuring device,repeatability and reproducibility of the system, the difference in loading conditions between the reference lab and how the system is being used, environmental conditions, and the difference in adapters. All of these can drastically increase the overall accuracy specification.

As a rule, accuracy is influenced by how the system is used, the frequency of calibration, the non-linearity of both the load cell and indicator, and thermal characteristics. In addition, the reference lab achieves short-term accuracy and does not include the system's stability or adapters, often the most significant error sources.

Several manufacturers claim specifications that use higher-order math equations for Non-Linearity to achieve unrealistic specifications, especially when programming an indicator with these values. At Morehouse, we find the button or washer-type load cells to have specifications that are difficult to meet.

The figure above shows an example of a 2-pt span calibration. Values are programmed at 1,000 and 10,000 lbf. These values can often be entered into the indicator or captured during setup with the force-measuring system under load. In the above example, you can see the instrument's bias or error. Instrument bias is defined as the average of replicate indications minus a reference quantity value. [i]

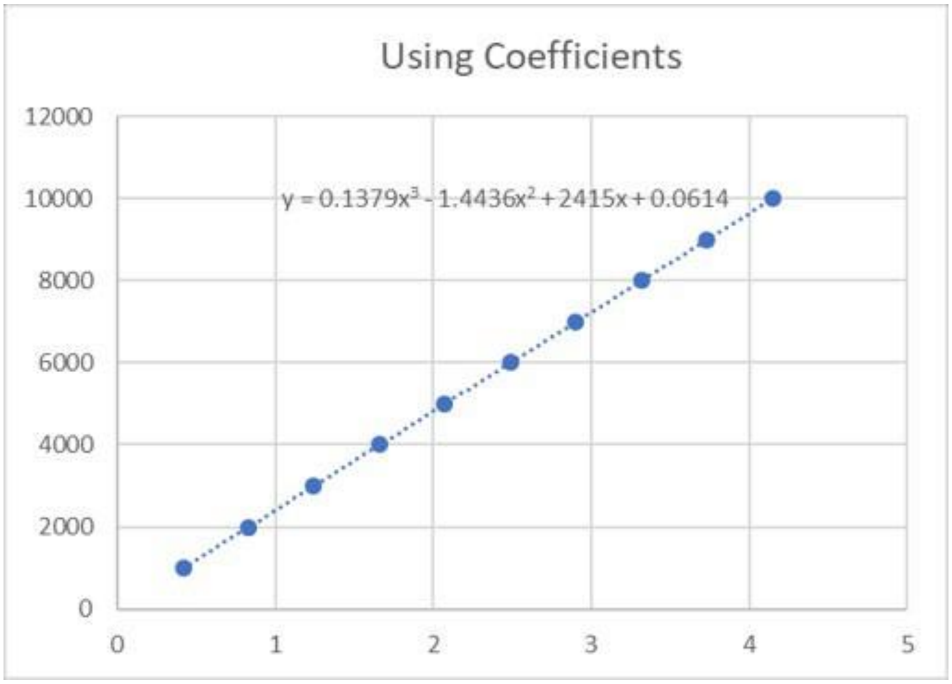
When discussing bias, we discuss the difference between the calculated and applied force values. In the example above, the worst error is 3.2 lbf, around 0.08 % of the applied force when 4,000 lbf is applied.

Many indicators do not allow the end-user to enter anything other than span points, and they do not allow using a polynomial equation with coefficients. Exceptions like the Morehouse C705P and 4215-Plus allow one to use points (data or span points) from the calibration data or coefficients.

Many of those indicators that do not allow a polynomial equation with coefficients to be used have USB, IEEE, RS232, or other interfaces that enable computers to read and communicate with the indicator. When software can communicate with an indicator, there may still be a way to use the polynomial equation using the coefficients for real-time response, converting the response to force to comply with ASTM E74 and ISO 376 requirements. ASTM E74 and ISO 376 require calibration to calculate higher-order polynomials, so the least squares method is often used.

Using Least Squares to Calculate Polynomial Equations

A polynomial equation is fitted to the calibration data using the least squares method to predict deflection values. The term "least squares" is used because it is the smallest sum of squares of errors. This method will contain a formula that is a bit more complex than a straight line, as most force-measuring devices will not be linear. A straight-line fit would be the best if a device were almost perfectly linear. The formula often uses higher-order polynomial equations to minimize errors and best replicate the line. The ASTM E74 standard goes into more detail on these equations and avoids overfitting and round-off errors if there is insufficient precision. The figure below shows a plot from the actual readings in mV/V and fits to a 3rd-order equation.



Graph of a 3rd Order Least Squares Fit.

Instead of using the equation for a straight line (y=mx+b), we have two formulas to solve for both force and response. These are:

Response (mV/V) = $A_0 + A_1(\text{Force}) + A_2(\text{Force})^2 + A_3(\text{Force})^{3,}$ and

Force (lbf) = $B_0 + B_1(\text{Response}) + B_2(\text{Response})^2 + B_3(\text{Response})^3$

When substituting these values with those in the equation shown on the line above, we are solving for force when we know the response; we would use $B_0 = 0.0614$, $B_1 = 2415$, $B_2 = -1.4436$, $B_3 = 0.17379$, so the formula becomes:

Force(lbf) = $0.0614 + 2415(\text{Response}) + -1.4436(\text{Response})^2 + 0.1379\,(\text{Response})^3$.

These are often called coefficients and are labeled as A_0, A_1, etc., and B_0, B_1, etc.; A_0 or B_0 would determine the point at which the equation crosses the Y-intercept, while the other coefficients determine the curve.

Many force standards allow curve fitting of a 3rd degree and limit the maximum degree fit to a 5th degree. The most recognized legal metrology standards for using coefficients are ASTM E74, primarily used in North America, and ISO 376, used throughout most of Europe and the rest of the world.

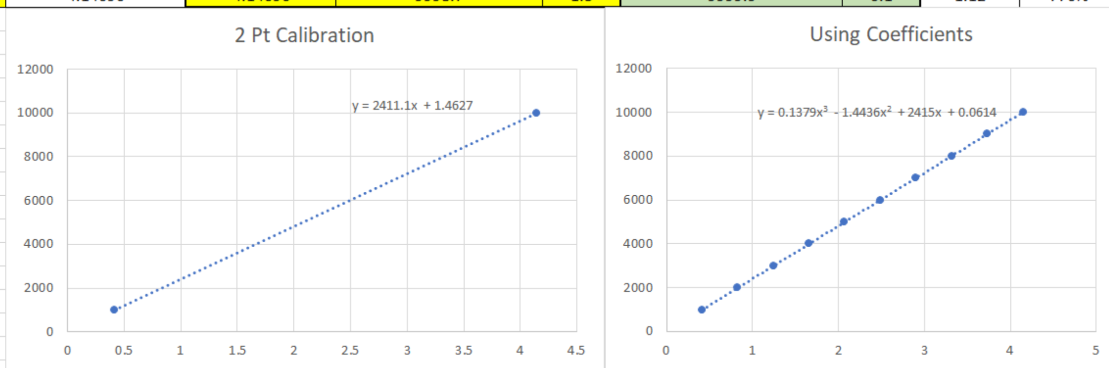
When the equation in the graph above is used on the actual readings, the values calculated using the coefficients are close to the applied force values. Thus, the bias, or measurement error, is around 0.1 lbf, far less than the 3.2 lbf error shown using a 2-pt span calibration.

Using Coefficient Conversion			
Calculated Values polynomial	Error	Diff in Errors	% difference
199.9	0.1	0.25	189%
999.9	0.1	-0.11	116%
1999.9	0.1	2.26	1846%
2999.9	0.1	2.82	2109%
3999.9	0.1	3.06	2413%
4999.9	0.1	3.05	2180%
5999.9	0.1	2.83	2060%
6999.9	0.1	2.47	1856%
7999.9	0.1	2.02	1446%
8999.9	0.1	1.56	1055%
9999.9	0.1	1.12	776%

Bias or Measurement Error When Using Coefficients.

The overall difference in the errors between these two methods is high. The figure below best summarizes these errors. One process produces an almost exact match, which is 0.001 % of full scale, while the other is 0.032 %. The worst point, at 4,000 lbf, has a difference of 3.06 lbf, or a 2413 % difference between errors. Using coefficients often requires additional software and a computer, whereas the 2-pt adjustment will not.

		Indicator with 2-pt adjustments			Using Coefficient Conversion			
Applied Force lbf	Actual Readings (mV/V)	Programmed Points	Calculated Values 2 pt span	Error	Calculated Values polynomial	Error	Diff in Errors	% difference
200	0.08279		199.6	0.4	199.9	0.1	0.25	189%
1000	0.41415	0.41415	1000.0	0.0	999.9	0.1	-0.11	116%
2000	0.82851		1997.6	2.4	1999.9	0.1	2.26	1846%
3000	1.24302		2997.0	3.0	2999.9	0.1	2.82	2109%
4000	1.65767		3996.8	3.2	3999.9	0.1	3.06	2413%
5000	2.07242		4996.8	3.2	4999.9	0.1	3.05	2180%
6000	2.48726		5997.0	3.0	5999.9	0.1	2.83	2060%
7000	2.90216		6997.4	2.6	6999.9	0.1	2.47	1856%
8000	3.31709		7997.8	2.2	7999.9	0.1	2.02	1446%
9000	3.73203		8998.3	1.7	8999.9	0.1	1.56	1055%
10000	4.14696	4.14696	9998.7	1.3	9999.9	0.1	1.12	776%



Difference Between 2-pt Span and Coefficients on the Same Load Cell.

TUR

$$\mathbf{TUR} = \frac{\textbf{Span of the } \pm \textbf{ UUT Tolerance}}{\mathbf{2 \ x \ k_{95\%}}\textbf{(Expanded Uncertainty of the Measurement Process)}}$$

TUR is a ratio of the tolerance of the item being calibrated divided by the uncertainty of the entire calibration process. Evaluation of the TUR is a rigorous process that includes additional contributors to the uncertainty beyond just the uncertainty of the calibration standard. ANSI/NCSLI Z540.3 and the Handbook published in 2006 completely define TUR. It relies on knowing how to calculate uncertainty following a calibration hierarchy, including metrological traceability. There is a lot of confusion in the industry on how to calculate Test Uncertainty Ratio.

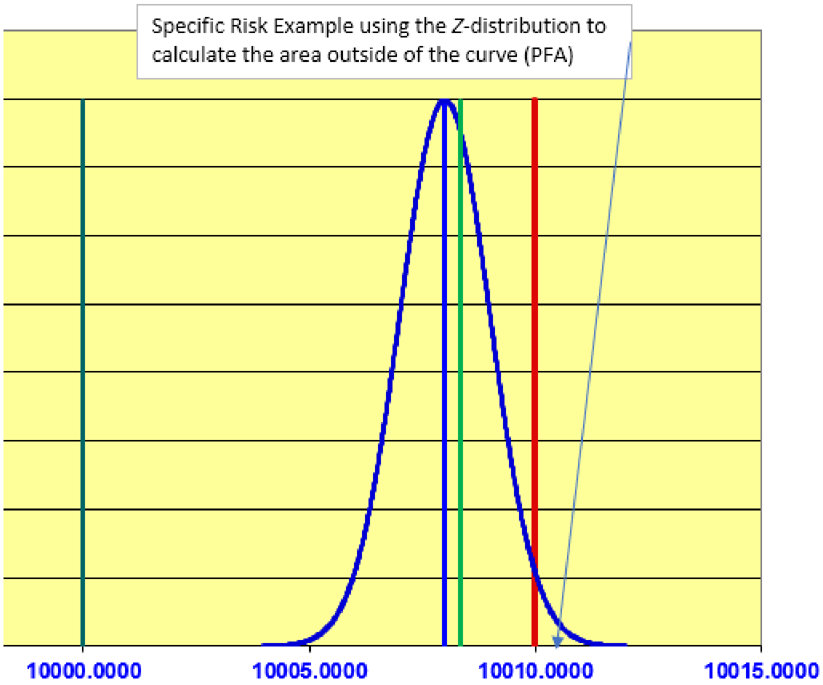
Rightfully so, the definition has taken different shapes and forms over the decades. Sometimes, it is even confused with the Test Accuracy Ratio (TAR).

$$\mathbf{TUR} = \frac{\textbf{Span of the } \pm \textbf{Tolerance}}{\mathbf{2\ x\ k_{95\%}}\left(\sqrt{\left(\frac{\mathbf{CMC}}{\mathbf{k_{CMC}}}\right)^2 + \left(\frac{\mathbf{Resolution_{UUT}}}{\sqrt{\mathbf{12}}}\right)^2 + \left(\frac{\mathbf{Repeatability_{UUT}}}{\mathbf{1}}\right)^2 + \cdots (\mathbf{u}_{Other})^2}\right)}$$

The TUR formula is an adaptation with the denominator clarified for current practices from Handbook for the Application of ANSI/NCSLI ANSI 540.3 -2006. Some may contend that resolution is accounted for with repeatability studies. However, if repeatability equals zero, then the UUT's resolution must be considered.

In most cases, the numerator is the UUT Accuracy Tolerance. The denominator is slightly more complicated. Per the ANSI/NCSL Z540.3 Handbook, "For the denominator, the 95 % expanded uncertainty of the measurement process used for calibration following the calibration procedure is to be used to calculate TUR. The value of this uncertainty estimate should reflect the results that are reasonably expected from using the approved procedure to calibrate the M&TE. Therefore, the estimate includes all components of error that influence the calibration measurement results, including the influences of the item being calibrated, except for the bias of the M&TE. The calibration process error, therefore, includes temporary and non-correctable influences incurred during the calibration, such as repeatability, resolution, error in the measurement source, operator error, error in correction factors, environmental influences, etc."

PFA for Specific Risk – Load Cell Example



To Calculate PFA, the Excel function is NORM.DIST.

Risk upper = NORM.DIST(Measured value, Upper Tolerance Limit, Standard Uncertainty, TRUE)

Risk Lower = 1- NORM.DIST(Measured value, Lower Tolerance Limit, Standard Uncertainty, TRUE)

PFA = Risk upper +Risk Lower

Load Cell Example

What would this document be without a force example for **Specific Risk**?

A customer sent their 10,000 N load cell in for calibration. The purchase order indicates calibration to the manufacturer's specification.

Since the purchase order is incomplete regarding pass/fail criteria and how measurement uncertainty is taken into account the customer is contacted and presented with several options based on their risk requirements.

The customer decides to rewrite the order. The new purchase order reads calibrate using a tolerance of 0.1 % of full scale (± 10 N) taking measurement uncertainty ($U_{95.45\ \%}$) into account using specific risk calculations. Fail if the PFA for either side > 2.5 %, otherwise pass.

Step 1 Calibrate the equipment; we will need to determine the Standard Uncertainty (k =1) of the Measurement Process for this calibration.

For simplistic sake, we will look at the 10,000 N point.

10,000 N force was applied three times, and the instrument read 10,000, 10,002, 10,001.

Taking the standard deviation of these numbers =stdev(10,000 10,002 10,001) we get 1

The resolution of the equipment is 1 N.

The CMC of the reference standard is 0.2 N.

$$\left(\sqrt{\left(\frac{\mathbf{CMC}}{\mathbf{k_{CMC}}}\right)^2 + \left(\frac{\mathbf{Resolution_{UUT}}}{\sqrt[2]{\mathbf{12}}}\right)^2 + \left(\frac{\mathbf{Repeatability_{UUT}}}{\mathbf{1}}\right)^2 + \cdots (\mathbf{u}_{Other})^2}\right)$$

Thus, the formula for Standard Uncertainty of the Measurement Process becomes.

$$\left(\sqrt{\left(\frac{0.2}{2}\right)^2 + \left(\frac{1}{\sqrt[2]{12}}\right)^2 + \left(\frac{1}{1}\right)^2}\right) = 1.04563 \text{ N}$$

We now have everything we need to calculate Guard Banded Acceptance Limits and PFA.

A 10,000 N load cell has a tolerance of ± 0.1 % of full scale.

The measured value is 10,000 N.

Upper tolerance = 10,010 N.

Lower Tolerance = 9,990 N.

Measured Value = 10,001 N.

Standard uncertainty = 1.04563 N.

Step 2 Calculate Acceptance Limits

We are calculating our Conformance probability for 97.50 % Confidence for symmetrical tolerances. We calculate the Guard band Multiplier by using the formula in Excel of **NORM.S.INV (0.975)/2.**

We then use this number of 0.98 as our GB Multiplier as follows.

*For the Guard band upper limit, we have 10010 – (GB Multiplier * Coverage Factor * Standard Measurement Uncertainty)*

*10010 – (0.980 * (2 *1.04563)) = 10007.9506*

*For the Guard band lower limit, we have 9990 + (GB Multiplier * Coverage Factor *Standard Measurement Uncertainty)*

*9990 + (0.980 * (2 * 1.04563)) = 9992.0494*

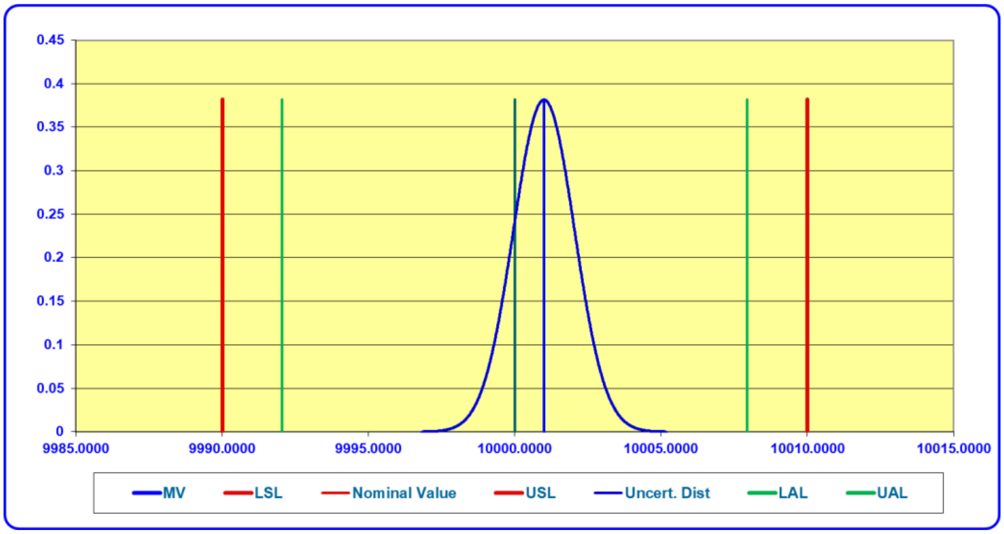


Figure 1: Graph showing the GB Acceptance Limits to limit PFA to 2.5 %

Thus, our acceptance limit is between *9992.0494 and 10007.9506, as any measured value between these two values will have less than 2.5 % PFA.*

Step 3 Calculate PFA

Risk Upper = NORM.DIST(10001, 10010, 1.04563, TRUE) = 0 %

Risk Lower = 1- NORM.DIST(10001, 9990, 1.04563, TRUE)) = 0 %

Total Risk = 0 %

Additional Proof

One can use the Upper or Lower GB Acceptance Limit to verify the GB acceptance limits.

Risk Upper = NORM.DIST(10007.950603, 10010, 1.04563, TRUE) = 2.5 %

Conclusion

Morehouse has been manufacturing force products for one hundred years. Several calculations have changed for the better and might change again. This document was made in hopes of simplifying what we do to make things easier for you to reproduce similar results. Not everything is captured in this document, though it is a great starting point.

Stay informed with the latest insights and tips on metrology by signing up for weekly updates from Henry. Visit https://mhforce.com/new-force-calibration-ebook-2024-edition/ to subscribe and receive tips and resources directly in your mailbox.

Annex (Sample Calculation of TUR)

Example: A customer sends a **10,000 lbf** load cell for calibration with an accuracy specification of **± 0.05 % of full scale**. The calibration provider uses a Universal Calibrating Machine to perform the calibration. When **10,000 lbf** is applied, the unit reads **10,001 lbf**. The display resolution is **1 lbf**.

Step 1: Calculate the numerator.

$$TUR = \frac{\text{Span of the} \pm \text{Tolerance}}{2 \times k_{95\%}\left(\sqrt[2]{\left(\frac{CMC}{k_{CMC}}\right)^2 + \left(\frac{Resolution_{UUT}}{\sqrt[2]{12}}\right)^2 + \left(\frac{Repeatability_{UUT}}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 11: TUR Formula Nominator

The device is a **10,000 lbf** load cell with an accuracy specification of **± 0.05 %**

10,000 * 0.0005 = ± 5 lbf

The upper specification limit is 10,000 + 5 = 10,005 lbf

The lower specification limit is 10,000 – 5 = 9,995 lbf

Therefore, the Span of the ±Tolerance is 10,005 – 9,995 = 10 lbf

$$TUR = \frac{10\ lbf}{2\ x\ k_{95\%}\left(\sqrt[2]{\left(\frac{CMC}{k_{CMC}}\right)^2 + \left(\frac{Resolution_{UUT}}{\sqrt[2]{12}}\right)^2 + \left(\frac{Repeatability_{UUT}}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 12: TUR Formula with the Numerator added.

Step 2: Calculate the denominator.

Everything is calculated to **1 standard deviation (Standard Uncertainty)** for this calculation.
Calibration and Measurement Capability (CMC)

$$TUR = \frac{\text{Span of the} \pm \text{Tolerance}}{2 \times k_{95\%}\left(\sqrt[2]{\left(\frac{CMC}{k_{CMC}}\right)^2 + \left(\frac{Resolution_{UUT}}{\sqrt[2]{12}}\right)^2 + \left(\frac{Repeatability_{UUT}}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 13: CMC portion of the denominator

CMC is the uncertainty at the calibrated force. The Universal Calibrating Machine has an uncertainty of **0.02 % at 10,000 lbf**.

The CMC is **10,000 * 0.0002 = 2 lbf**

k_{CMC} **is 2**, which was listed on the calibration provider's certificate.

Dividing the CMC by 2, the standard uncertainty is reported at **one standard deviation**. In most cases, the **CMC uncertainty component is reported at approximately 95 %**, and **a coverage factor of** $k = 2$ is used.

$$TUR = \frac{10\ lbf}{2\ x\ k_{95\%}\left(\sqrt[2]{\left(\frac{2\ lbf}{2}\right)^2 + \left(\frac{Resolution_{UUT}}{\sqrt[2]{12}}\right)^2 + \left(\frac{Repeatability_{UUT}}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 14: TUR Formula with CMC added

UUT Resolution

$$TUR = \frac{\text{Span of the} \pm \text{Tolerance}}{2\,x\,k_{95\%}\left(\sqrt[2]{\left(\frac{CMC}{k_{CMC}}\right)^2 + \left(\frac{Resolution_{UUT}}{\sqrt[2]{12}}\right)^2 + \left(\frac{Repeatability_{UUT}}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 15: Resolution portion of the denominator

Resolution$_{UUT}$ for force instrument is calculated by dividing the force applied by the output at applied force and then multiplying this by the instrument's readability.

The **Resolution**$_{UUT}$ is **(10,000 lbf / 10,000 lbf) * 1 = 1 lbf**

To convert **1 lbf** resolution to standard uncertainty, it is either divided by the **square root of 12**, or the square root of 3 depending on the Type of resolution.

$$TUR = \frac{10\ lbf}{2\ x\ k_{95\%}\left(\sqrt[2]{\left(\frac{2\ lbf}{2}\right)^2 + \left(\frac{1\ lbf}{\sqrt[2]{12}}\right)^2 + \left(\frac{Repeatability_{UUT}}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 16: TUR Formula with Resolution added.

Repeatability

$$TUR = \frac{\text{Span of the} \pm \text{Tolerance}}{2 \times k_{95\%}\left(\sqrt[2]{\left(\frac{CMC}{k_{CMC}}\right)^2 + \left(\frac{Resolution_{UUT}}{\sqrt[2]{12}}\right)^2 + \left(\frac{Repeatability_{UUT}}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 17: Repeatability portion of the denominator

For this example, **five replicate readings** are taken.

Repeatability is obtained by applying a force of **10,000 lbf** to the **Unit Under Test (UUT)** five times, and the sample standard deviation of five replicated measurements is calculated.

Repeatability of sample size five: **(10,000, 10,001, 10,000, 10,001, 10,001) = 0.54772**
Since the repeatability is already expressed as one standard deviation, the divisor is 1.

$$TUR = \frac{10\ lbf}{2\ x\ k_{95\%}\left(\sqrt[2]{\left(\frac{2\ lbf}{2}\right)^2 + \left(\frac{1\ lbf}{\sqrt[2]{12}}\right)^2 + \left(\frac{0.54772}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 18: TUR Formula with Repeatability added.

Other Error Sources

$$TUR = \frac{\text{Span of the} \pm \text{Tolerance}}{2 \times k_{95\%}\left(\sqrt[2]{\left(\frac{CMC}{k_{CMC}}\right)^2 + \left(\frac{Resolution_{UUT}}{\sqrt[2]{12}}\right)^2 + \left(\frac{Repeatability_{UUT}}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 19: Other error sources in the denominator

Other error sources attributed to the **CPU** can be considered for the **UUT**. Some examples are environmental influences, error in correction factors, etc. For this example, other error sources are inherent in repeatability and **CMC**.

$$TUR = \frac{10\ lbf}{2\ x\ k_{95\%}\left(\sqrt[2]{\left(\frac{2\ lbf}{2}\right)^2 + \left(\frac{1\ lbf}{\sqrt[2]{12}}\right)^2 + \left(\frac{0.54772}{1}\right)^2}\right)}$$

Figure 20: TUR Formula with all error sources added.

Calculate the Denominator

Sum of all the contributors = **SQRT((2/2)^2+(1/3.464)^2+(0.54772/1)^2) = 1.1762**

$$\textbf{TUR} = \frac{10\ lbf}{2\ x\ k_{95\%}\ (1.1762)}$$

Figure 21: TUR Calculated

The specification of **10 lbf** is divided by: **2 * k at 95 %** Calibration Process Uncertainty (**k= 2** for this example)

$$\textbf{TUR} = \frac{10\ lbf}{2\ x\ 2.35231} \qquad \textbf{TUR} = \frac{10\ lbf}{4.70462}$$

Figure22: TUR Calculated

TUR = 2.1256

References

JCGM 106:2012_E clause 3.3.15 "Evaluation of measurement data – The role of measurement uncertainty in conformity assessment."

ISO/IEC 17025:2017 "General requirements for the competence of testing and calibration laboratories."

ASTM E74-18 Standard Practices for Calibration and Verification for Force-Measuring Instruments

ANSI/NCSL Z540.3 Handbook "Handbook for the Application of ANSI/NCSLI 540.3-2006 - Requirements for the Calibration of Measuring and Test Equipment."

ILAC P-14:09/2020, "Policy for Uncertainty in Calibration," clause 5.4

ISO 376:2011 Metallic materials — Calibration of force-proving instruments used for the verification of uniaxial testing machines